Paper 2 Option J

Further Mechanics 1 Mark Scheme (Section A)

Question	Scheme	Marks	AOs
1(a)	Using the model and $v^2 = u^2 + 2as$ to find v	M1	3.4
	$v^2 = 2as = 2g \times 2.4 = 4.8g \Rightarrow v = \sqrt{(4.8g)}$	A1	1.1b
	Using the model and $v^2 = u^2 + 2as$ to find u	M1	3.4
	$0^2 = u^2 - 2g \times 0.6 \Rightarrow u = \sqrt{(1.2g)}$	A1	1.1b
	Using the correct strategy to solve the problem by finding the sep. speed and app. speed and applying NLR	M1	3.1b
	$e = \sqrt{(1.2g)} / \sqrt{(4.8g)} = 0.5$ *	A1*	1.1b
		(6)	
(b)	Using the model and e = sep. speed / app. speed, $v = 0.5\sqrt{(1.2g)}$	M1	3.4
	Using the model and $v^2 = u^2 + 2as$	M1	3.4
	$0^2 = 0.25\,(1.2g) - 2gh \Rightarrow h = 0.15$ (m)	A1	1.1b
		(3)	
(c)	Ball continues to bounce with the height of each bounce being a quarter of the previous one	B1	2.2b
		(1)	
			(10 marks)

Notes:

(a)
M1: For a complete method to find v
A1: For a correct value (may be numerical)
M1: For a complete method to find u
A1: For a correct value (may be numerical)
M1: For finding both v and u and use of Newton's Law of Restitution
A1*: For the given answer

(b)
M1: For use of Newton's Law of Restitution to find rebound speed
M1: For a complete method to find h
A1: For 0.15 (m) oe

(c)
B1: For a clear description including reference to a quarter

Question	Scheme	Marks	AOs
2(a)	Energy Loss = KE Loss – PE Gain	M1	3.3
	$= \frac{1}{2} \times 0.5 \times 25^2 - 0.5\,g \times 20$	A1	1.1b
	$= 58.25 = 58$ (J) or 58.3 (J)	A1	1.1b
		(3)	
(b)	Using work-energy principle, $20\,R = 58.25$	M1	3.3
	$R = 2.9125 = 2.9$ or 2.91	A1ft	1.1b
		(2)	
(c)	Make resistance variable (dependent on speed)	B1	3.5c
		(1)	
			(6 marks)

Notes:

(a)
M1: For a difference in KE and PE
A1: For a correct expression
A1: For either 58 (2sf) or 58.3(3sf)

(b)
M1: For use of work-energy principle
A1ft: For either 2.9 (2sf) or 2.91 (3sf) follow through on their answer to (a)

(c)
B1: For variable resistance oe

Question	Scheme	Marks	AOs
3(a)	Force = Resistance (since no acceleration) = 30	B1	3.1b
	Power = Force × Speed = 30 × 4	M1	1.1b
	= 120 W	A1 **ft**	1.1b
		(3)	
(b)	Resolving parallel to the slope	M1	3.1b
	$F - 60g\sin\alpha - 30 = 0$	A1	1.1b
	$F = 70$	A1	1.1b
	Power = Force × Speed = 70 × 3	M1	1.1b
	= 210 W	A1 **ft**	1.1b
		(5)	
			(8 marks)

Notes:

(a)
B1: For force = 30 seen
M1: For use of $P = Fv$
A1ft: For 120 (W), follow through on their '30'

(b)
M1: For resolving parallel to the slope with correct no. of terms and 60*g* resolved
A1: For a correct equation
A1: For $F = 70$
M1: For use of $P = Fv$
A1ft: For 210 (W), follow through on their '70'

Question	Scheme	Marks	AOs
4(a)	Use of conservation of momentum	M1	3.1a
	$3mu - 2mu = 3mv + mw$	A1	1.1b
	Use of NLR	M1	3.1a
	$3ue = -v + w$	A1	1.1b
	Using a correct strategy to solve the problem by setting up two equations (need both) in u and v and solving for v	M1	3.1b
	$v=\frac{u}{4}(1 - 3e)$	A1	1.1b
		(6)	
(b)	$\frac{u}{4}(1 - 3e) < 0$	M1	3.1b
	$\frac{1}{3} < e \leq 1$	A1	1.1b
		(2)	
(c)	Solving for w	M1	2.1
	$w=\frac{u}{4}(1 + 9e)$ *	A1 *	1.1b
		(2)	
(d)	Substitute $e=\frac{5}{9}$	M1	1.1b
	$v = -\frac{u}{6}, w=\frac{3u}{2}$	A1	1.1b
	Use NLR for impact with wall, $x = fw$	M1	1.1b
	Further collision if $x > -v$	M1	3.4
	$f\frac{3u}{2} > \frac{u}{6}$	A1	1.1b
	$1 \geq f > \frac{1}{9}$	A1	1.1b
		(6)	
			(16 marks)

Notes:

(a)
M1: For use of CLM, with correct no. of terms, condone sign errors
A1: For a correct equation
M1: For use of Newton's Law of Restitution, with e on the correct side
A1: For a correct equation
M1: For setting up *two* equations and solving their equations for v
A1: For a correct expression for v

(b)
M1: For use of an appropriate inequality
A1: For a complete range of values of e

(c)
M1: For solving their equations for w
A1: For the given answer

Question 4 notes continued:
(d)
M1: For substituting $e = \frac{5}{9}$ into their v and w
A1: For correct expressions for v and w
M1: For use of Newton's Law of Restitution, with e on the correct side
M1: For use of appropriate inequality
A1: For a correct inequality
A1: For a correct range

Further Mechanics 2 Mark Scheme (Section B)

Question	Scheme	Marks	AOs
5 (a)	Multiply out and differentiate wrt t	M1	1.1b
	$v = 3t^2 - 16t + 20 \Rightarrow a = 6t - 16$	A1	1.1b
		(2)	
(b)	Multiply out and integrate wrt t	M1	1.1b
	$s = \int 3t^2 - 16t + 20 \mathrm{d}t = t^3 - 8t^2 + 20t (+C)$	A1	1.1b
	$t = 0, s = 0 \Rightarrow C = 0$ $t = 2, s = 8 - 32 + 40 = 16$	A1	1.1b
		(3)	
(c)	$s = 0 \Rightarrow t^3 - 8t^2 + 20t = 0$ and $t \neq 0 \Rightarrow t^2 - 8t + 20 = 0$	M1	2.1
	Explanation to show that $t^2 - 8t + 20 > 0$ for all t.	M1	2.4
	So $s = 0$ has no non-zero solutions, so s is never zero again, so never returns to O *	A1*	3.2a
		(3)	
			(8 marks)

Notes:

(a)
M1: For multiplying out and differentiating (powers decreasing by 1)
A1: For a correct expression for a

(b)
M1: For multiplying out and integrating (powers increasing by 1)
A1: For a correct expression for s with or without C
A1: For $C = 0$ and correct final answer

(c)
M1: For equating their s to 0 and producing a quadratic
M1: For clear explanation that $t^2 - 8t + 20 > 0$ for all t (e.g. completing the square or another complete method)
A1*: For a correct conclusion in context

Question	Scheme	Marks	AOs
6(a)	$\cos\alpha = \frac{4}{5}$ or $\sin\alpha = \frac{3}{5}$	B1	1.1b
	$r = 4a\sin\alpha$	B1	1.1b
	Resolving vertically	M1	3.1b
	$T_1\cos\alpha - T_2\sin\alpha = mg$	A1	1.1b
	Resolving horizontally	M1	3.1b
	$T_1\sin\alpha + T_2\cos\alpha = mr\omega^2$	A1	1.1b
	$T_1\sin\alpha + T_2\cos\alpha = mr\omega^2$	A1	1.1b
	Solving for either tension	M1	2.1
	$T_1 = \frac{4m}{25}(9a\omega^2 + 5g)$ *	A1*	1.1b
	$T_2 = \frac{3m}{25}(16a\omega^2 - 5g)$ *	A1*	1.1b
		(10)	
(b)	$\frac{4m}{25}(9a\omega^2 + 5g) < 4mg$	M1	2.1
	$\frac{3m}{25}(16a\omega^2 - 5g) > 0$	M1	2.1
	$\omega > \sqrt{\frac{5g}{16a}}$ or $\omega < \sqrt{\frac{20g}{9a}}$	A1	2.2a
	$S = \frac{2\pi}{\omega}$	M1	1.1b
	$3\pi\sqrt{\frac{a}{5g}} < S < 8\pi\sqrt{\frac{a}{5g}}$ *	A1*	1.1b
		(5)	
(c)	String being light implies that the tension is constant in both portions of the string	B1	3.5b
		(1)	
			(16 marks)

Notes:

(a)
B1: For correct trig. ratio seen
B1: For a correct radius expression seen
M1: For resolving vertically with correct no. of terms and tensions resolved
A1: For a correct equation
M1: For resolving horizontally with correct no. of terms and tensions resolved
A1A1: For a correct equation
M1: For solving their two equations to find either tension
A1*: For the given answer
A1*: For the given answer

Question 6 notes continued:
(b) **M1:** For use of $T_1 < 4mg$ **M1:** For using $T_2 > 0$ **A1:** For a correct inequality (either) for ω **M1:** For use of $S = \frac{2\pi}{\omega}$ with either critical value **A1*:** For given answer
(c) **B1:** For a clear explanation

Question	Scheme	Marks	AOs
7(a)	Rel. Mass: 2 5 1 8	B1	1.2
	y : 2 0.5 1.5 $\bar{y}$	B1	1.2
	x : 0.5 2.5 4.5 $\bar{x}$	B1	1.2
	$(2 \times 2) + (5 \times 0.5) + (1 \times 1.5) = 8\bar{y}$	M1	2.1
	$\bar{y} = 1$ *	A1*	1.1b
	$(2 \times 0.5) + (5 \times 2.5) + (1 \times 4.5) = 8\bar{x}$	M1	2.1
	$\bar{x} = 2.25$	A1	1.1b
		(7)	
(b)	Use of correct strategy to solve the problem by use of 'moments equation'	M1	3.1b
	$(8 \times 2.25) - (2\pi r^2 \times 0.5) = (8 - 2\pi r^2)2.5$	A1ft	1.1b
	Solving for r	M1	1.1b
	$r = \frac{1}{\sqrt{2\pi}} = 0.399$	A1	1.1b
		(4)	
(c)	Since $\bar{y}$ for original plate is 1, holes must be symmetrically placed about the line $y = 1$	B1	2.4
	$a = 1.5$	B1	2.2a
		(2)	
(d)	Use of tan from an appropriate triangle	M1	1.1a
	$\tan\alpha = \frac{2}{1.5} = \frac{4}{3}$	A1ft	1.1b
	$\alpha = 53.1^\circ$	A1	1.1b
		(3)	
			(16 marks)

Notes:

(a)
B1: For correct relative masses
B1: For correct y values
B1: For correct x values
M1: For a moments equation, correct no. of terms, condone sign errors
A1*: For a correct given answer (1)
M1: For a moments equation, correct no. of terms
A1: For 2.25

(b)
M1: For a moments equation, correct no. of terms, condone sign errors
A1ft: For a correct equation, follow through on their $\bar{x}$
M1: For solving for r
A1: For 0.399 or 0.40

Question 7 notes continued:
(c) **B1:** For consideration of symmetry about $y = 1$ **B1:** For $a = 1.5$
(d) **M1:** For use of tan from an appropriate triangle **A1ft:** For a correct equation, follow through on their a **A1:** For a correct angle